Finding Limits Analytically

1. Limits of continuous functions can be evaluated with direct substitution.

   a. $\lim\limits_{x\to -5}[3x^2 + 4x - 5]$ b. $\lim\limits_{p\to -2} \dfrac{2p+4}{3p}$ ~~c. $\lim\limits_{x\to 0}[3e^x - \sin x + \ln(x+1)]$~~

2. Use factoring and/or expanding, then use cancellation

   a. $\lim\limits_{x\to -2} \dfrac{2x+4}{x^2-3x-10}$

   b. $\lim\limits_{x\to -3} \dfrac{x^3+27}{x+3}$ (Hint: Factor sum of cubes)

   c. $\lim\limits_{t\to 1} \dfrac{2t^2+3t-5}{1-t}$

   d. $\lim\limits_{h\to 0} \dfrac{(-1+h)^2-1}{h}$

   e. $\lim\limits_{h\to 0} \dfrac{(3+h)^3-27}{h}$

3. Use our "rationalizing the denominator/numerator" technique

   a. $\lim\limits_{x\to -10} \dfrac{\sqrt{x+19}-3}{x+10}$

   b. $\lim\limits_{h\to 0} \dfrac{\sqrt{4+h}-2}{h}$

   c. $\lim\limits_{x\to 1} \dfrac{1-\sqrt{x}}{x-1}$

4. Solve by simplifying the compound fraction

   a. $\lim\limits_{h\to 0} \dfrac{\frac{1}{1+h}-1}{h}$

   b. $\lim\limits_{x\to -4} \dfrac{\frac{8}{x+6}-4}{x+4}$

   c. $\lim\limits_{x\to 2} \dfrac{2-x}{\frac{2}{x}-1}$

5. Limits involving trigonometric functions

   a. $\lim\limits_{x\to 0} \dfrac{\sin x}{2x}$

   b. $\lim\limits_{t\to 0} \dfrac{\sin 3t}{5t}$

   c. $\lim\limits_{x\to 0} \dfrac{\tan 4x}{x}$

Find Limits Analytically   Solutions to study guide

1a. $\lim_{x\to -5} 3x^2+4x-5 = 3(-5)^2+4(-5)-5$
$= 75-20-5 = \boxed{50}$

1b. $\lim_{p\to -2} \frac{2p+4}{3p} = \frac{2(-2)+4}{3(-2)} = \frac{-4+4}{-6} = 0$

2a. $\lim_{x\to -2} \frac{2x+4}{x^2-3x-10} = \frac{-4+4}{4+6-10} = \frac{0}{0}$ ind.

$\therefore f(x) = \frac{2\cancel{(x+2)}}{\cancel{(x+2)}(x-5)} = \frac{2}{x-5}$; $\lim_{x\to -2} f(x) = \frac{2}{-2-7}$
$= \boxed{-\frac{2}{7}}$

2b. $\lim_{x\to -3} \frac{x^3+27}{x+3} = \frac{-27+27}{-3+3} = \frac{0}{0}$ ind.

$\therefore f(x) = \frac{\cancel{(x+3)}(x^2-3x+9)}{\cancel{(x+3)}}$; $\therefore \lim_{x\to -3} f(x) = 9+9+9 = 27$

2c.) $\lim_{t\to 1} \frac{2t^2+3t-5}{1-t} = \frac{2+3-5}{1-1} = \frac{0}{0}$ ind.

2c.) $\frac{2t^2+3t-5}{1-t} = \frac{(2t+5)\cancel{(t-1)}}{-\cancel{(t-1)}} = -(2t+5)$ 2

$\lim_{t \to 0} f(t) = -(2(0)+5) = -5$

2d.)
$\lim_{h \to 0} \frac{(h+-1)^2-1}{h} = \frac{1-1}{0} = \frac{0}{0}$, ind.

$f(x) = \frac{h^2-2h+1-1}{h} = \frac{h^2-2h}{h} = h-2$

$\therefore \lim_{h \to 0} f(x) = 0-2 = -2$

2e.) $\frac{\lim_{h \to 0}(3+h)^3-27}{h} = \frac{3^3-27}{0} = \frac{0}{0}$ ind.

$f(x) = \frac{(3+h)^3-27}{h} = 27+(3^2h)3+3(3h^2)+h^3$

$= \frac{27+27h+9h^2+h^3-27}{h}$

1
1 1
1 2 1
1 3 3 1

$\lim_{h \to 0} f(x) = \underbrace{= (27+9h+h^2)h/h}_{27+9h+h^2} = 27+9(0)+(0)^2 = 27$

3a. $\lim\limits_{x\to -10} \dfrac{\sqrt{x+19}-3}{\sqrt{x+10}} = \dfrac{3-3}{0} = \dfrac{0}{0}$ ind. 13

$$f(x) = \frac{(\sqrt{x+19}-3)(\sqrt{x+19}+\sqrt{3})}{\sqrt{x+10}\,(\sqrt{x+19}+\sqrt{3})}$$

$$= \frac{(x+19-9)}{\sqrt{x+10}\,(\sqrt{x+19}+\sqrt{3})}$$

$$= \frac{(x+10)^1}{(x+10)^{1/2}(\sqrt{x+19}+\sqrt{3})} = \frac{(x+10)^{1/2}}{\sqrt{x+19}+\sqrt{3}}$$

$$\lim_{x\to -10} f(x) = \frac{\sqrt{0}}{\sqrt{3}+\sqrt{3}} = \frac{0}{2\sqrt{3}} = 0$$

3b. $\lim\limits_{h\to 0} \dfrac{\sqrt{4+h}-2}{h} = \dfrac{\sqrt{4}-2}{0} = \dfrac{2-2}{0}$ = ind.

$$f(h) = \frac{\sqrt{4+h}-2}{h}\cdot\frac{\sqrt{h+4}+2}{\sqrt{h+4}+2} = \frac{h+4-4}{h((h+4)^{1/2}+2)} = \frac{h}{h((h+4)^{1/2}+2)}$$

$$= \frac{1}{\sqrt{h+4}+2};\quad \lim_{h\to 0} f(h) = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

3c.) $\frac{1-\sqrt{x}}{x-1}$; $\lim\limits_{x\to 1} f(x) = \frac{1-1}{1-1} = \frac{0}{0}$ ind

$$\frac{(1-\sqrt{x})(1+\sqrt{x})}{(x-1)(1+\sqrt{x})} = \frac{1-x^2}{(x-1)(1+x^{1/2})} = \frac{-(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(x^{1/2}+1)}$$

$f(x) = \frac{-(x+1)}{x^{1/2}+1}$; $\lim\limits_{x\to 1} f(x) = \frac{-2}{1+1} = -1$

4a. $\lim\limits_{h\to 0} \frac{\frac{1}{1+h} - 1}{h} = \frac{\frac{1}{1} - 1}{0} = \frac{1-1}{0} = \frac{0}{0}$ ind

$$f(x) = \frac{\frac{1}{1+h} - 1}{h} = \frac{1-(1+h)}{1+h} \div h$$

$$= \frac{1-1-h}{1+h} \cdot \frac{1}{h} = \frac{-\cancel{h}}{1+h} \cdot \frac{1}{\cancel{h}}$$

$\lim\limits_{h\to 0} \frac{-1}{h+1} = -1$

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4b.) $\displaystyle\lim_{x\to -4} \frac{\frac{8}{x+6} - 4}{x+4} = \frac{4-4}{-4+4} = \frac{0}{0}$ ind.

$$\frac{\frac{8}{x+6} - \frac{4(x+6)}{x+6}}{x+4} = \frac{8 - 4x - 24}{(x+6)(x+4)} = \frac{-4x-16}{(x+4)(x+6)}$$

$$= \frac{-4\cancel{(x+4)}}{\cancel{(x+4)}(x+6)}$$

$$\lim_{x\to -4} f(x) = \frac{-4}{-4+6} = \frac{-4}{2} = -2$$

4c.) $\displaystyle\lim_{x\to 2} \frac{2-x}{\frac{2}{x} - 1} = \frac{2-2}{\frac{2}{2}-1} = \frac{0}{0}$ ind.

$$f(x) = \frac{2-x}{\frac{2}{x}-1} = \frac{(2-x)}{\left[\frac{2-x}{x}\right]} = \frac{\cancel{2-x}}{1} \cdot \frac{x}{\cancel{(2-x)}}$$

$$\lim_{x\to 2} f(x) = 2$$

5a. $\displaystyle\lim_{x\to 0} \frac{\sin x}{2x} = \lim_{x\to 0} \frac{1}{2} \frac{\sin x}{x} = \frac{1}{2} \cdot 1 = \frac{1}{2}$

5b. $\displaystyle\lim_{t\to 0} \frac{\sin 3t}{5t} = \lim_{t\to 0} \frac{3}{5} \cdot \frac{\sin 3t}{3t} = \frac{3}{5} \cdot 1 = \frac{3}{5}$

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5c.) $\lim_{x\to 0} \frac{\tan 4x}{x} = \lim_{x\to 0} \frac{\sin 4x}{x} \cdot \frac{1}{\cos 4x}$

$= \lim_{x\to 0} \frac{4}{4} \cdot \frac{\sin 4x}{x} \cdot \frac{1}{\cos 4x}$

$= \lim_{x\to 0} 4 \cdot \frac{\sin 4x}{4x} \cdot \frac{1}{\cos 4x} = 4 \cdot 1 \cdot 1 = 4$

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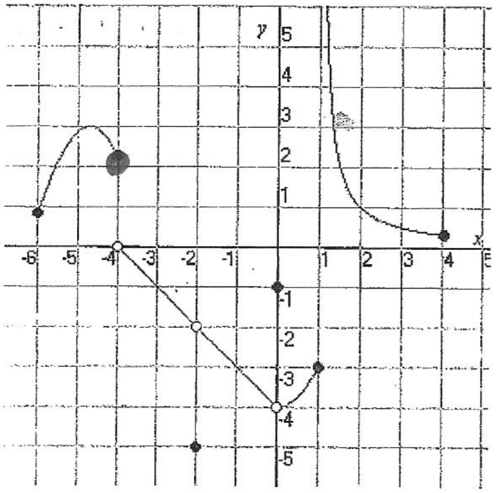
33. Given the following graph below of $f(x)$, find the following:



(a) $\lim_{x\to -4^-} f(x) = 2$

(b) $\lim_{x\to -2} f(x) = -2$

(c) $f(-2) = -5$

(d) $\lim_{x\to 1^-} f(x) = -3$

(e) $\lim_{x\to 1^+} f(x) = \infty$

(f) $\lim_{x\to 1} f(x) =$ DNE

(g) $f(1) = -3$

(h) $\lim_{x\to 2} f(x) = 1$

Use the figure at right to answer the following questions:

34. $\lim_{x\to 3} f(x) = 3$

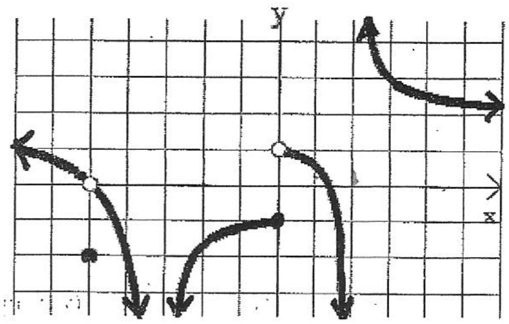
35. $\lim_{x\to \infty} f(x) = 2$

36. $\lim_{x\to 2^+} f(x) = \infty$

37. $\lim_{x\to 0} f(x) =$ DNE

38. $\lim_{x\to -\infty} f(x) = 1$

39. $\lim_{x\to -5} f(x) = 0$



Graph of f

6. $\lim_{x\to\infty} \tan x =$ DNE

7. $\lim_{x\to -\infty} \sin x =$ DNE

8. $\lim_{x\to -\infty} -x^2+1 = -\infty$

9. $\lim_{x\to \infty} x^3+x^2-1 = \infty$

10. $\lim_{x\to -\infty} \frac{x}{x+1} = 1$

11. $\lim_{x\to \infty} \frac{-x}{2x^2+1} = 0$

12. $\lim_{x\to -\infty} \frac{x^2}{x+1} = -\infty$