Name Key Period ______ Date ____________________

Rational Functions Study Guide (Algebra Boot Camp) Honors Calculus Perry

Part I: For each rational function, find all the holes, intercepts, and asymptotes.
Find also the domain, and do a sign analysis. Use each to sketch the graph of the function.

SHOW YOUR WORK ON A SEPARATE SHEET OF PAPER.

1. $f(x) = \frac{x+3}{-3x+12} = \frac{x+3}{-3(x+4)}$

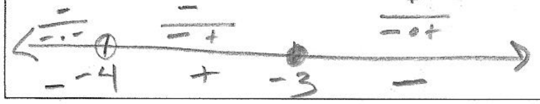
Holes: None

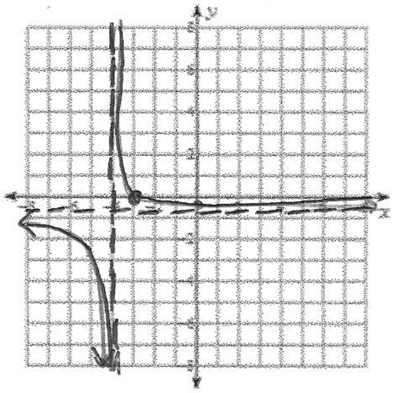
Intercepts: $(-3, 0), (0, -1/4)$

Asymptotes: $y = \frac{-1}{3}$; $x = -4$

Domain: $\{x \mid x \neq -4\}$

Sign Analysis:





$\frac{-7}{-5} = 1.2$

2. $f(x) = \frac{x^2+x-12}{x^2+5x+4} = \frac{(x-3)\cancel{(x+4)}}{(x+1)\cancel{(x+4)}} = \frac{x-3}{x+1}$

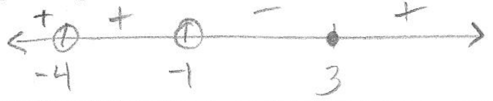
Holes: $(-4, 7/5)$

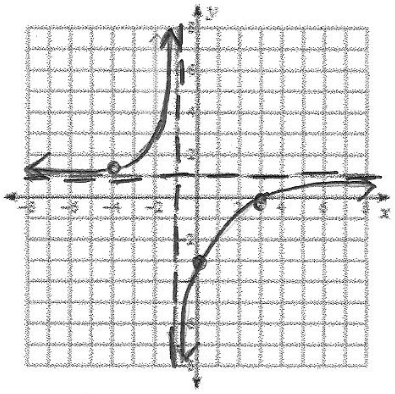
Intercepts: $(3, 0), (0, -3)$

Asymptotes: $x = -1$, $y = 1$

Domain: $\{x \mid x \neq -4, -1\}$

Sign Analysis:





3. $f(x) = \frac{-x^2+x}{x^2+x-6} = \frac{-x(x-1)}{(x-2)(x+3)}$

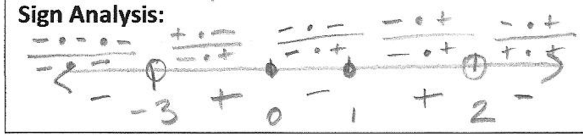
Holes: none

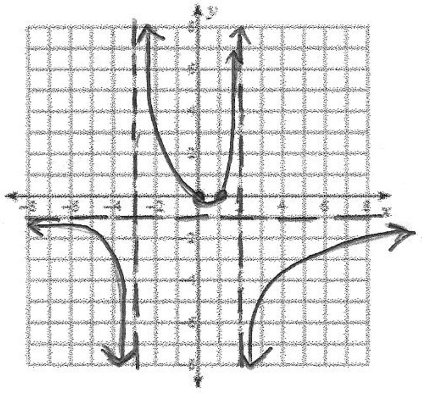
Intercepts: (0,0), (1,0)

Asymptotes: $x = 2, -3$; $y = -1$

Domain: $\{x \mid x \neq -3, 2\}$

Sign Analysis:





4. $f(x) = \frac{x^3-4x}{3x^2-3x} = \frac{x(x^2-4)}{3x(x-1)} = \frac{\cancel{x}(x+2)(x-2)}{3\cancel{x}(x-1)}$

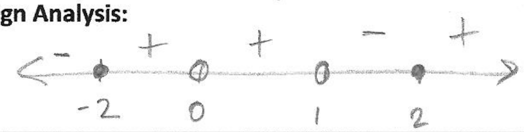
Holes: $\left(0, \frac{4}{3}\right)$

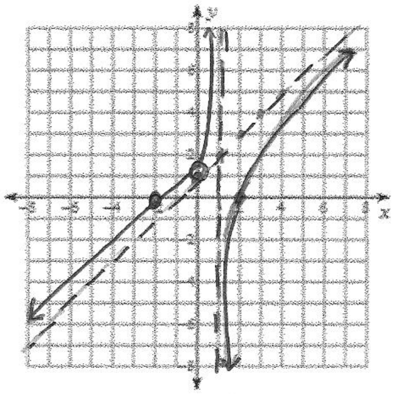
Intercepts: (2,0), (-2,0)

Asymptotes: $x = 1$; $y = x+1$

Domain: $\{x \mid x \neq 1\}$

Sign Analysis:





SA: $\frac{x^2-4}{x-1} \rightarrow$

$$\begin{array}{r|rrr} 1 & 1 & 0 & -4 \\ & & 1 & 1 \\ \hline & 1 & 1 & -3 \end{array}$$

$x+1 = y$

5. $f(x) = \frac{x+2}{x^2+4}$

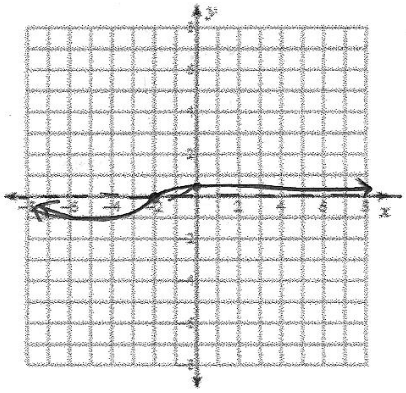
Holes: none

Intercepts: $(0,\frac{1}{2}), (-2,0)$

Asymptotes: $y=0$

Domain: $\mathbb{R}$

Sign Analysis:



6. $f(x) = \frac{4x^2}{x^2+4}$

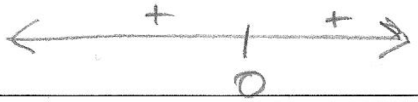
Holes: none

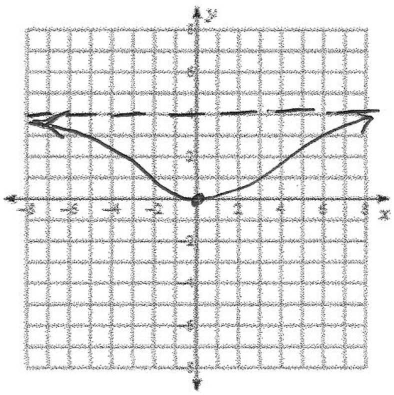
Intercepts: $(0,0)$

Asymptotes: $y=4$

Domain: $\mathbb{R}$

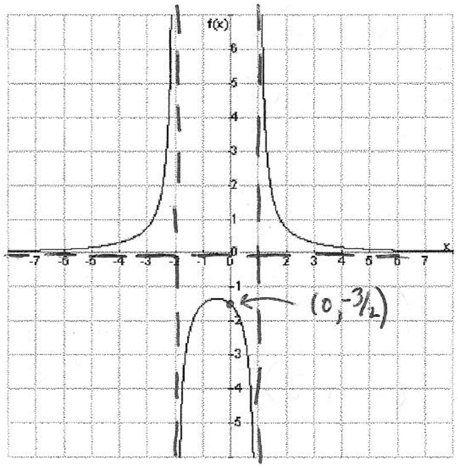
Sign Analysis:





Part II: For the graph of each rational function, use what you know about holes, intercepts, asymptotes, and multiplicity to write a possible equation for the graph. Without using a graphing calculator, explain how you know that the equation is accurate.

1.



y=0
x=-2
x=2

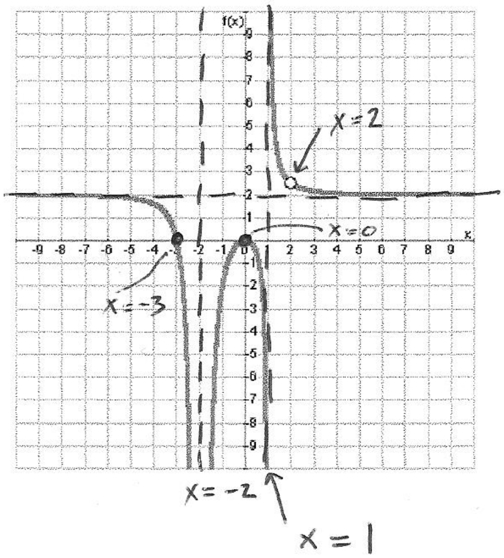
F(x) = $\dfrac{6}{(x+2)(x-2)}$

Explain: I saw it had asymptotes of $x=\pm 2$, so I knew the factors were $(x+2)$ & $(x-2)$. Also, the HA was $y=0$, so the numerator could be a number only. (If it took the form $x-a$, there would be an intercept.) To Figure out the value of the number on top, I picked the y-intercept $(0,-3/2)$, substituted, & solved for the numerator, A.

$$\frac{A}{(x+2)(x-2)} = \frac{-3}{2}$$

$$\frac{A}{-4} = \frac{-3}{2}; A=6$$

2.



y=2

F(x) = $\dfrac{2(x+3)(x)^2(x-2)}{(x+2)^2(x-1)\,(x-2}$

Explain: First, I have intercepts of $x=-3$, $x=0$, so they are numerators on top of $(x+3)$ & (x), respectively. Next I have a hole @ $x=2$, so the factor on top & bottom is $(x-2)$. Next, I have asymptotes $x=-2$ & $x=1$, so the factors in the denominator are $(x+2)$ & $(x-1)$, respectively. Next, the function touches at $x=0$, so its multiplicity is even; I chose 2. At $x=-3$, the function crosses, so its multiplicity is odd; I chose 1. Also, the function "comes together" on either side of the asymptote $x=-2$, so its factor, $(x+2)$, also has →

even multiplicity; I chose 2. Because the function does not "come together" on either side of $x=1$, its factor has an odd multiplicity; I chose 1. Finally, the horizontal asymptote is $y=2$; so I put a two in the numerator as a factor. Thus, the function is

$$y = \frac{2x^2(x-2)(x+3)}{(x-1)(x-2)(x+2)^2}$$