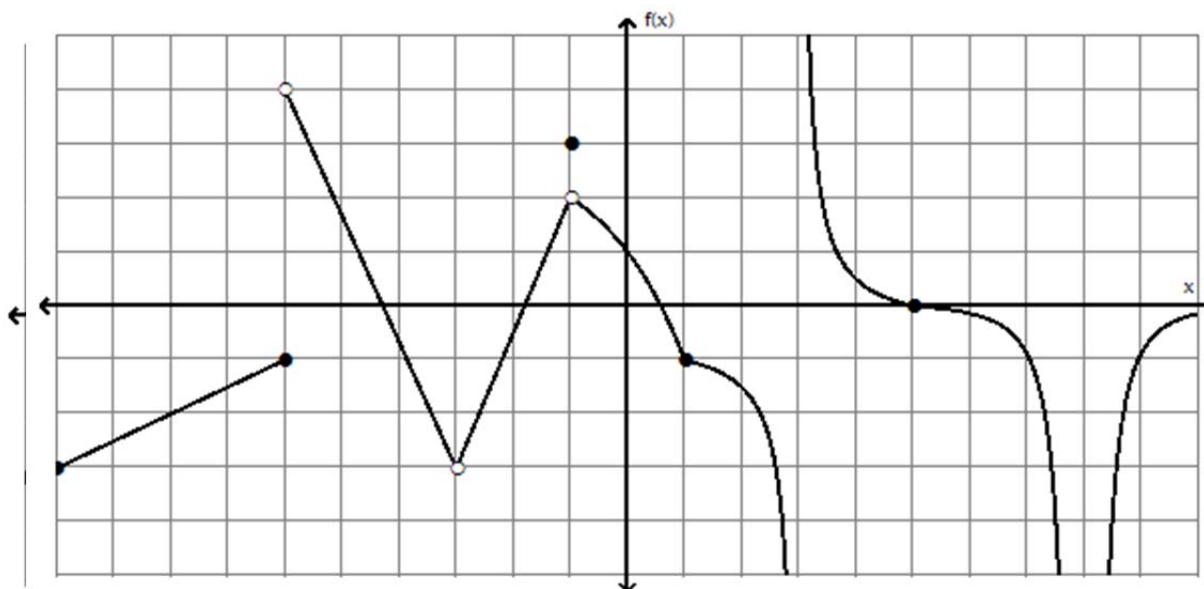


# Introduction To Limits

Name Key



Use the graph above to evaluate each limit, or if appropriate, indicate that the limit does not exist.

- |    |                                  |                       |     |                                 |                             |
|----|----------------------------------|-----------------------|-----|---------------------------------|-----------------------------|
| 1. | $\lim_{x \rightarrow -6^-} f(x)$ | <b>-1</b>             | 8.  | $\lim_{x \rightarrow -1} f(x)$  | <b>2</b>                    |
| 2. | $\lim_{x \rightarrow -6^+} f(x)$ | <b>4</b>              | 9.  | $\lim_{x \rightarrow 1} f(x)$   | <b>-1</b>                   |
| 3. | $\lim_{x \rightarrow -6} f(x)$   | <b>does not exist</b> | 10. | $\lim_{x \rightarrow 3^-} f(x)$ | <b><math>-\infty</math></b> |
| 4. | $\lim_{x \rightarrow -3^+} f(x)$ | <b>-3</b>             | 11. | $\lim_{x \rightarrow 3^+} f(x)$ | <b><math>+\infty</math></b> |
| 5. | $\lim_{x \rightarrow -3} f(x)$   | <b>-3</b>             | 12. | $\lim_{x \rightarrow 3} f(x)$   | <b>does not exist</b>       |
| 6. | $\lim_{x \rightarrow -1^-} f(x)$ | <b>2</b>              | 13. | $\lim_{x \rightarrow 8^+} f(x)$ | <b><math>-\infty</math></b> |
| 7. | $\lim_{x \rightarrow -1^+} f(x)$ | <b>2</b>              | 14. | $\lim_{x \rightarrow 8} f(x)$   | <b><math>-\infty</math></b> |

## Solution to the Sudoku With Limits Puzzle

$$A = 3$$

$$B = 9$$

$$C = 8$$

$$D = 5$$

$$E = 9$$

$$F = 3$$

$$G = 2$$

$$H = 7$$

$$I = 4$$

$$J = 6$$

$$K = 5$$

$$L = 3$$

$$M = 6$$

$$N = 8$$

$$O = 4$$

$$P = 7$$

$$Q = 3$$

$$R = 8$$

$$S = 1$$

$$T = 3$$

$$U = 9$$

$$V = 4$$

$$W = 4$$

$$X = 7$$

$$Y = 1$$

$$Z = 6$$

$$a = 9$$

$$b = 1$$

3	5	8	9	6	7	4	1	2
1	6	4	8	2	5	7	9	3
2	9	7	3	4	1	8	6	5
9	2	1	5	7	4	3	8	6
6	8	5	2	9	3	1	4	7
7	4	3	6	1	8	5	2	9
8	1	6	7	3	2	9	5	4
4	7	9	1	5	6	2	3	8
5	3	2	4	8	9	6	7	1

## Calculus 1 Worksheet #4

**Limits involving trigonometric functions:**  $\lim_{x \rightarrow 0} \frac{\sin(\square)}{\square}$

### KNOW THE FOLLOWING THREE THEOREMS:

A. $\lim_{x \rightarrow 0} \frac{\sin \square}{\square} = 1$	B. $\lim_{x \rightarrow 0} \frac{\square}{\sin \square} = 1$	C. $\lim_{x \rightarrow 0} \frac{1 - \cos \square}{\square} = 0$
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### Examples:

1. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \cdot \left[ \frac{3}{3} \right] \Rightarrow \lim_{x \rightarrow 0} 3 \left[ \frac{\sin 3x}{3x} \right] = \boxed{3}$
2. $\lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x} \cdot \left[ \frac{7}{7} \right] \Rightarrow \lim_{x \rightarrow 0} 7 \left[ \frac{1 - \cos 7x}{7x} \right] = \boxed{0}$
3. $\lim_{x \rightarrow 0} \frac{\tan 2x}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 2x}{\cos 2x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 2x}{x \cos 2x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 2x}{x \cos 2x} \cdot \left[ \frac{2}{2} \right] \Rightarrow$ $\lim_{x \rightarrow 0} \frac{2}{\cos 2x} \left[ \frac{\sin 2x}{2x} \right] \Rightarrow \lim_{x \rightarrow 0} \frac{2}{\cos 2x} \Rightarrow \lim_{x \rightarrow 0} \frac{2}{\cos 2(0)} = \boxed{2}$

### Problems:

1. $\lim_{x \rightarrow 0} \frac{\sin \frac{1}{2}x}{x}$	2. $\lim_{x \rightarrow 0} x \csc x$	3. $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin x}$	4. $\lim_{x \rightarrow 0} \frac{\sin ax}{x}, a \neq 0$
5. $\lim_{x \rightarrow 0} \frac{\tan x}{x}$	6. $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x}$	7. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$	8. $\lim_{x \rightarrow 0} \frac{\sin x}{2x}$
9. $\lim_{x \rightarrow 0} \frac{3 \sin x}{x}$	10. $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$	11. $\lim_{x \rightarrow 0} \frac{\sin 4x}{2x}$	12. $\lim_{x \rightarrow 0} \frac{3x}{\sin x}$
13. $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$	14. $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$	15. $\lim_{x \rightarrow 0} \frac{\sin^4 2x}{4x^4}$	16. $\lim_{x \rightarrow 0} \frac{\sin 5x}{5x}$
17. $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{2x}$	18. $\lim_{x \rightarrow 0} \frac{x+2}{\cos x}$	19. $\lim_{x \rightarrow \frac{\pi}{4}} (\tan x)$	20. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$

### Answers:

1) $\frac{1}{2}$	2) 1	3) 2	4) a	5) 1	6) $\frac{3}{2}$	7) 3	8) $\frac{1}{2}$
9) 3	10) $\frac{3}{5}$	11) 2	12) 3	13) 0	14) $\frac{a}{b}$	15) 4	16) 1
17) 0	18) 2	19) 1	20) $\frac{1}{2}$				