

Introduction To Limits

Name _____



Use the graph above to evaluate each limit, or if appropriate, indicate that the limit does not exist.

1. $\lim_{x \rightarrow -6^-} f(x)$

8. $\lim_{x \rightarrow -1} f(x)$

2. $\lim_{x \rightarrow -6^+} f(x)$

9. $\lim_{x \rightarrow 1} f(x)$

3. $\lim_{x \rightarrow -6} f(x)$

10. $\lim_{x \rightarrow 3^-} f(x)$

4. $\lim_{x \rightarrow -3^+} f(x)$

11. $\lim_{x \rightarrow 3^+} f(x)$

5. $\lim_{x \rightarrow -3} f(x)$

12. $\lim_{x \rightarrow 3} f(x)$

6. $\lim_{x \rightarrow -1^-} f(x)$

13. $\lim_{x \rightarrow 8^+} f(x)$

7. $\lim_{x \rightarrow -1^+} f(x)$

14. $\lim_{x \rightarrow 8} f(x)$

Sudoku Puzzle with Limits

Solve the 28 limit problems below and place the answer in the corresponding cell (labeled A, B, C, ... Y, Z, a, b). Your answers should be integers from 1 to 9 inclusive. Then solve the resulting SUDOKU puzzle.

The rules of Sudoku are simple.

Enter digits from 1 to 9 into the blank spaces.

Every row must contain one of each digit.

So must every column, and so must every 3x3 square.

Each Sudoku has a unique solution that can be reached logically without guessing.

A. $\lim_{x \rightarrow 1} 3x$

B. $\lim_{x \rightarrow 2} (2x + 5)$

C. $\lim_{x \rightarrow 2} (2x^2 - 2x + 4)$

D. $\lim_{x \rightarrow 3} \frac{x+2}{x-2}$

E. $\lim_{x \rightarrow 7} (9)$

F. $\lim_{x \rightarrow 4} \frac{-4(x^2 - 11x + 28)}{x^2 - 4x}$

G. $\lim_{x \rightarrow \frac{\pi}{2}} 2 \tan\left(\frac{x}{2}\right)$

H. $\lim_{x \rightarrow -2} \left(\frac{x^2 - 4}{x^2 + 4} - \frac{14}{x} \right)$

I. $\lim_{h \rightarrow 0} \left(\frac{(2+h)^2 - 2^2}{h} \right)$

J. $\lim_{x \rightarrow 2} \left(\frac{4 - x^2}{3 - \sqrt{x^2 + 5}} \right)$

$$\text{K. } \lim_{k \rightarrow \infty} \left(\frac{10k - 2}{2k + 7} \right)$$

$$\text{L. } \lim_{x \rightarrow 4} \sqrt{25 - x^2}$$

$$\text{M. } \lim_{x \rightarrow \pi} \left(12 \sin \left(\frac{x}{6} \right) \right)$$

$$\text{N. } \lim_{p \rightarrow \infty} \left(\frac{16p^2 - 10p - 2}{5 - 2p + 2p^2} \right)$$

$$\text{O. } \lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{x - 2} \right)$$

$$\text{P. } \lim_{x \rightarrow 4} \left(\frac{x^2 - x - 12}{x - 4} \right)$$

$$\text{Q. } \lim_{x \rightarrow -1} \frac{2x^2 + 7x + 5}{x + 1}$$

$$\text{R. } \lim_{x \rightarrow 0} \frac{16(\sqrt{1+x} - 1)}{x}$$

$$\text{S. } \lim_{x \rightarrow -4} \frac{-3x - 23}{2x - 3}$$

$$\text{T. } \lim_{x \rightarrow 2} \left(\frac{x^2 - 1}{x - 1} \right)$$

$$\text{U. } \lim_{x \rightarrow 3} \left(\frac{2x^3 - 54}{x^2 - 9} \right)$$

$$\text{V. } \lim_{x \rightarrow -1} \left(\frac{2x^2 + 8x + 6}{x^2 + 3x + 2} \right)$$

$$\text{W. } \lim_{x \rightarrow 1} \left(\frac{2x - 2}{\sqrt{x^2 + 3} - 2} \right)$$

$$\text{X. } \lim_{x \rightarrow +\infty} ((x^2 + 5) + (2 - x^2))$$

$$\text{Y. } \lim_{x \rightarrow 4} \frac{x^2 - 7x + 12}{x - 4}$$

$$\text{Z. } \lim_{x \rightarrow 1} \left(\frac{\frac{3x}{1-x}}{\frac{1}{1-x^2}} \right)$$

$$\text{a. } \lim_{x \rightarrow \infty} \left(\frac{2 - 2x^2 + 9x^4}{x^4 + 3x^3 - 2x} \right)$$

$$\text{b. } \lim_{x \rightarrow 3} \frac{-x^2 + 7x - 12}{x - 3}$$

Calculus 1 Worksheet #4

Limits involving trigonometric functions: $\lim_{x \rightarrow 0} \frac{\sin(\square)}{\square}$

KNOW THE FOLLOWING THREE THEOREMS:

A. $\lim_{x \rightarrow 0} \frac{\sin \square}{\square} = 1$	B. $\lim_{x \rightarrow 0} \frac{\square}{\sin \square} = 1$	C. $\lim_{x \rightarrow 0} \frac{1 - \cos \square}{\square} = 0$
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Examples:

$1. \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \cdot \left[\frac{3}{3} \right] \Rightarrow \lim_{x \rightarrow 0} 3 \left[\frac{\sin 3x}{3x} \right] = \boxed{3}$
$2. \lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x} \cdot \left[\frac{7}{7} \right] \Rightarrow \lim_{x \rightarrow 0} 7 \left[\frac{1 - \cos 7x}{7x} \right] = \boxed{0}$
$3. \lim_{x \rightarrow 0} \frac{\tan 2x}{x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 2x}{\cos 2x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 2x}{x \cos 2x} \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 2x}{x \cos 2x} \cdot \left[\frac{2}{2} \right] \Rightarrow$ $\lim_{x \rightarrow 0} \frac{2}{\cos 2x} \left[\frac{\sin 2x}{2x} \right] \Rightarrow \lim_{x \rightarrow 0} \frac{2}{\cos 2x} \Rightarrow \lim_{x \rightarrow 0} \frac{2}{\cos 2(0)} = \boxed{2}$

Problems:

1. $\lim_{x \rightarrow 0} \frac{\sin \frac{1}{2}x}{x}$	2. $\lim_{x \rightarrow 0} x \csc x$	3. $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin x}$	4. $\lim_{x \rightarrow 0} \frac{\sin ax}{x}, a \neq 0$
5. $\lim_{x \rightarrow 0} \frac{\tan x}{x}$	6. $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x}$	7. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$	8. $\lim_{x \rightarrow 0} \frac{\sin x}{2x}$
9. $\lim_{x \rightarrow 0} \frac{3 \sin x}{x}$	10. $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$	11. $\lim_{x \rightarrow 0} \frac{\sin 4x}{2x}$	12. $\lim_{x \rightarrow 0} \frac{3x}{\sin x}$
13. $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$	14. $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$	15. $\lim_{x \rightarrow 0} \frac{\sin^4 2x}{4x^4}$	16. $\lim_{x \rightarrow 0} \frac{\sin 5x}{5x}$
17. $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{2x}$	18. $\lim_{x \rightarrow 0} \frac{x+2}{\cos x}$	19. $\lim_{x \rightarrow \frac{\pi}{4}} (\tan x)$	20. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$

Answers:

1) $\frac{1}{2}$	2) 1	3) 2	4) a	5) 1	6) $\frac{3}{2}$	7) 3	8) $\frac{1}{2}$
9) 3	10) $\frac{3}{5}$	11) 2	12) 3	13) 0	14) $\frac{a}{b}$	15) 4	16) 1
17) 0	18) 2	19) 1	20) $\frac{1}{2}$				