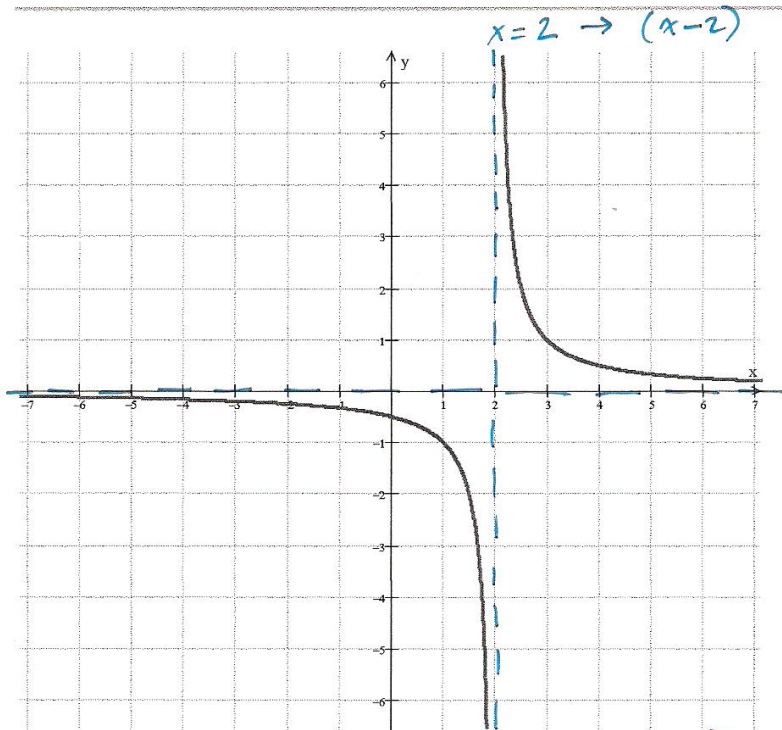


Name: Solutions

Period: _____ Date: _____

Algebra Bootcamp for Limits!
Rational Functions Backwards



$$f(x) = \frac{1}{x-2}$$

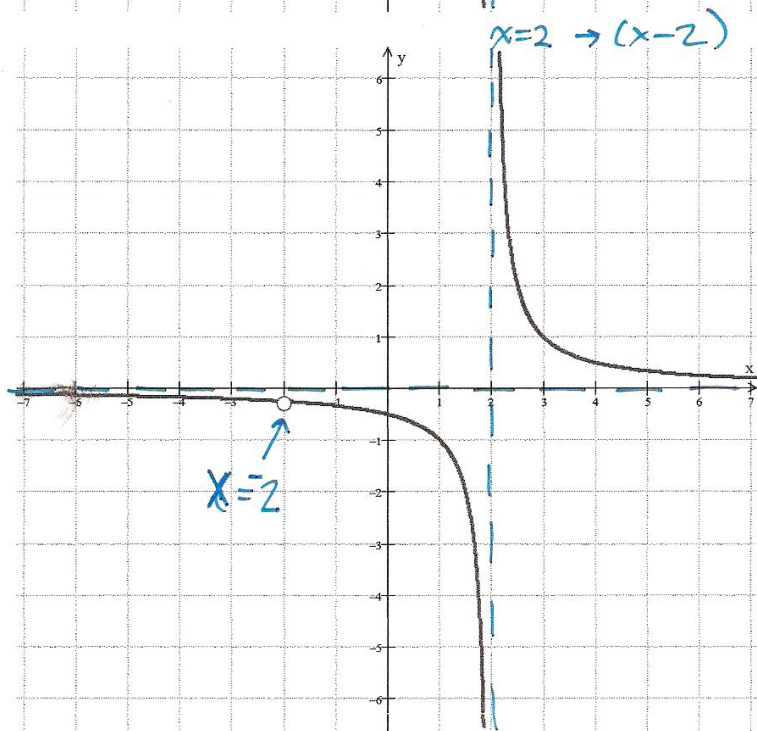
$\begin{array}{c} + \\ - \end{array}$
 \leftarrow
 $\begin{array}{c} | \\ - \end{array}$
 $\begin{array}{c} + \\ + \end{array}$
 \rightarrow

$\begin{array}{c} 2 \\ + \end{array}$

sign analysis matches graph

so, $f(x) = \frac{1}{x-2}$

$y=0$



$$f(x) = \frac{(x+2)}{(x+2)(x-2)}$$

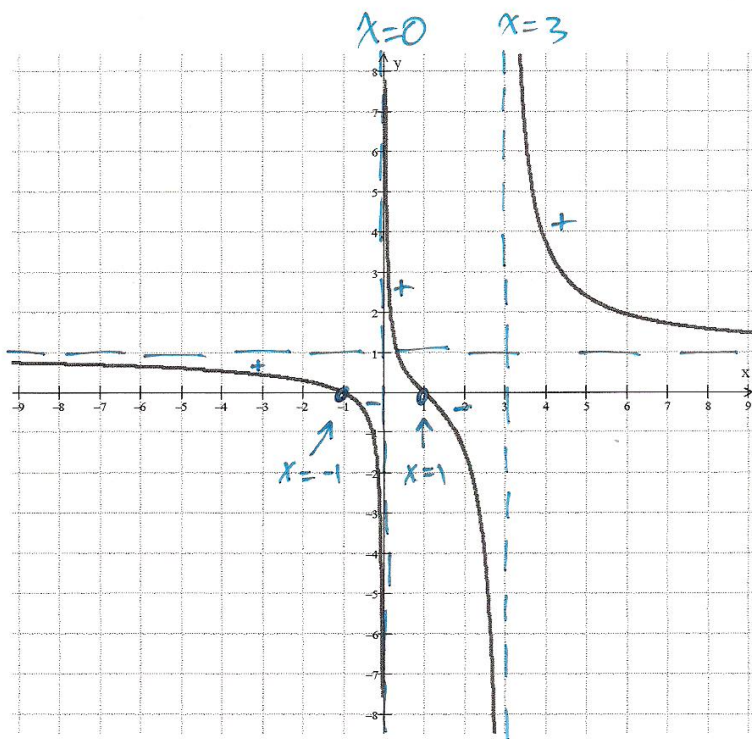
$\begin{array}{c} - \quad - \quad + \\ \leftarrow \quad \oplus \quad \quad \oplus \quad \rightarrow \\ -2 \quad \quad \quad 2 \end{array}$

sign analysis matches

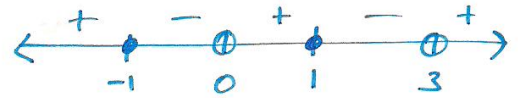
If $x = -2 \rightarrow$
 $y = \frac{1}{x-2} = \frac{1}{-2-2} = -\frac{1}{4}$

The hole appears to have the coordinates $(-2, -\frac{1}{4})$.

so $f(x) = \frac{(x+2)}{(x+2)(x-2)}$ or $\frac{x+2}{x^2-4}$



Multiplicities: $x = -1$ odd
 $x = 0$ odd
 $x = 1$ odd
 $x = 3$ odd

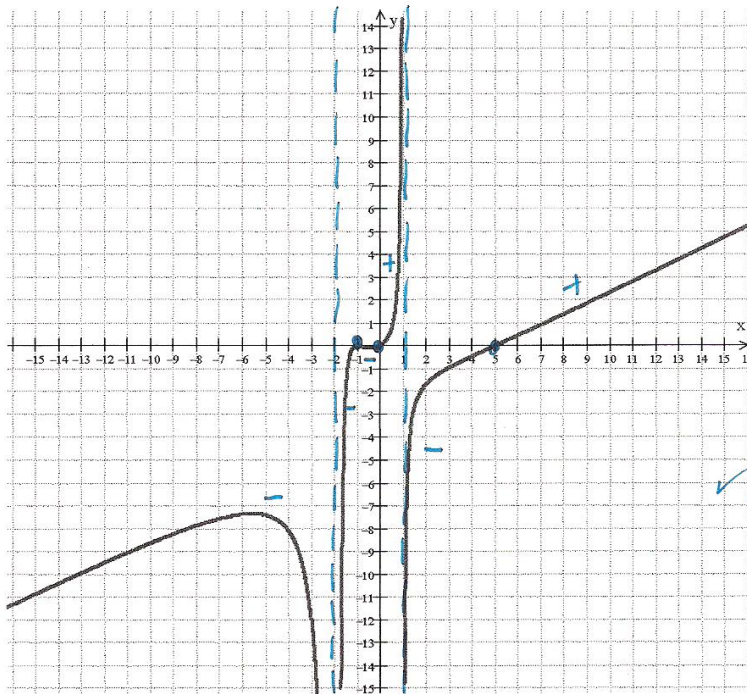


$$y = 1$$

$$f(x) = \frac{(x+1)(x-1)}{x(x-3)}$$

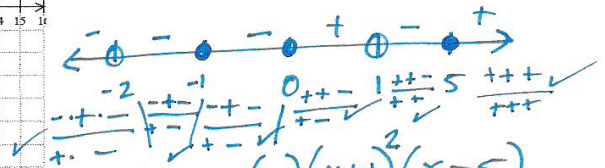
sign analysis for \uparrow matches.

$$\text{So: } f(x) = \frac{x^2 - 1}{x^2 - 3x}$$



Multiplicities:

$x = -2$ even
 $x = -1$ even
 $x = 0$ odd
 $x = 1$ odd
 $x = 5$ odd



$$f(x) = \frac{(x)(x+1)(x-5)}{(x+2)^2(x-1)}$$

signs match.

Degree ratio is: 4:3, so

SA resulted

$$\text{So: } f(x) = \frac{x(x-5)(x+1)^2}{(x+2)^2(x-1)}$$