Intercepts and holes

1.) $f(x) = \dfrac{x^2-1}{x^2-2x-3} = \dfrac{(x+1)(x-1)}{(x+1)(x-3)} = \dfrac{x-1}{x-3}$.

$0 = \dfrac{x-1}{x-3} \rightarrow 0 = x-1 \rightarrow x = 1 \rightarrow (1,0)$

$f(0) = \dfrac{0-1}{0-3} = \dfrac{1}{3} \rightarrow (0, 1/3)$

Intercepts: $(0, 1/3), (1,0)$

Hole: $x+1=0 \rightarrow x=-1$; $f(-1) = \dfrac{-1-1}{-1-3} = \dfrac{-2}{-4} = \dfrac{1}{2}$

$\therefore (-1, 1/2)$ is the hole

2.) $g(x) = \dfrac{x+2}{x-1}$

Intercepts: $x=0 \rightarrow g(0) = \dfrac{2}{-1} = -2 \rightarrow (0,-2)$

$y=0 \rightarrow 0 = \dfrac{x+2}{x-1} \rightarrow 0 = x+2 \rightarrow x = -2$

$\rightarrow (-2,0)$

$\therefore (-2,0)$ & $(0,-2)$ No Holes

3.) $R(x) = \dfrac{3x+3}{2x+4} = \dfrac{3(x+1)}{2(x+2)}$ No Holes

Intercepts: $x=0 \rightarrow \dfrac{3(1)}{2(2)} = \dfrac{3}{4} \rightarrow \left(0, \dfrac{3}{4}\right)$

$y=0 \rightarrow 0 = 3x+3 \rightarrow -3 = 3x \rightarrow x = -1$ $(-1,0)$

Intercepts

4.) $y = \dfrac{x-1}{x^2-9} = \dfrac{x-1}{(x+3)(x+-3)}$; Intercepts:

$x=0 \rightarrow \dfrac{-1}{-9} = \dfrac{1}{9} = y$

[No Holes]

[(0, 1/9)]

$y=0 \rightarrow 0 = x-1$

$1 = x \rightarrow$ [(1, 0)]

5.) $y = \dfrac{x^2+x-12}{x^2-4} = \dfrac{(x+4)(x-3)}{(x-2)(x+2)}$

[No Holes] Intercepts: $x=0 \rightarrow \dfrac{4(+3)}{+2(2)} = \dfrac{6}{2} = 3$

(0, 3)

$y=0 \rightarrow 0 = (x+4)(x+3)$

$x = -4, -3$

[(-3, 0), (-4, 0)
(0, 3)]

6.) $k(x) = \dfrac{25-x^2}{x+5} = \dfrac{-(x^2-25)}{x+5} = \dfrac{(x+5)(x-5)}{(x+5)}$

Hole: $x=5$, $y = -5-5 = -10$: (-5, -10) Hole

Ints: $x=0 \rightarrow \dfrac{25}{5} = 5$ (0, 5)

$y=0 \rightarrow 25 - x^2 \rightarrow x = \pm 5 \rightarrow$ (-5, 0), (5, 0) Ints

7.) $f(x) = \dfrac{x^2+x-12}{x-4} = \dfrac{(x-4)(x+3)}{(x-4)}$

Hole: $x = 4$; $f(x) = x+3$

$f(4) = 4+3 = 7 \rightarrow \boxed{(4,7)}$

Ints: $x=0 \rightarrow f(0) = 0+3 = 3 \rightarrow (0,3)$

$y=0 \rightarrow 0 = (x-4)(x+3)$

$x = 4, x = -3$

$\rightarrow \boxed{(-3,0), (0,3), (4,0)}$

8.) $y = \dfrac{3x^4+4}{x^3+3x} = \dfrac{(\sqrt{3}x^2)^2 + (2)^2}{x(x^2+3)}$

No Holes

$x=0 \rightarrow \dfrac{4}{0} \rightarrow$ und. #

$y=0 \rightarrow 3x^4+4 = 0 \rightarrow 3x^4 = -4$ #

No intercepts

9.) $f(x) = \dfrac{x+1}{x^2+1}$

No Holes

$x=0 \rightarrow f(0) = \dfrac{1}{1} = 1 \rightarrow (0,1)$

$y=0 \rightarrow 0 = x+1 \rightarrow x = -1 \rightarrow (-1,0)$

Intercepts

10.) $y = \dfrac{x^2-1}{x} = \dfrac{(x-1)(x+1)}{x}$

No Holes

$x=0 \rightarrow \dfrac{-1 \cdot 1}{0} \rightarrow$ und. #

$y=0 \rightarrow 0 = x^2-1$

$1 = x^2 \rightarrow x = \pm 1$

10 cont'd) Intercepts: $(-1,0), (1,0)$

No Holes