

Asymptotes & Domain

①

$$1.) \frac{x^2 - 1}{x^2 - 2x - 3} = f(x) = \frac{(x+1)(x-1)}{(x+1)(x-3)} = \frac{x-1}{x-3}$$

V. Asymptote: $x-3=0 \rightarrow \boxed{x=3}$

H. Asymptote: $\frac{x}{x} = 1 = y \rightarrow \boxed{y=1}$

Domain: $\{x \mid x \neq 3\}$

$$2.) g(x) = \frac{x+2}{x-1}$$

V. Asymptote: $x-1=0 \rightarrow \boxed{x=1}$

HA: $y = \frac{x}{x} = 1 = y \rightarrow \boxed{y=1}$

Domain: $\{x \mid x \neq 1\}$

$$3.) R(x) = \frac{3(x+1)}{2(x+2)}$$

VA: $x+2=0 \rightarrow \boxed{x=-2}$

HA: $y = \frac{3x}{2x} = \frac{3}{2} = y \rightarrow \boxed{y = \frac{3}{2}}$

Domain: $\{x \mid x \neq -2\}$

$$4.) y = \frac{x-1}{x^2-9} = \frac{x-1}{(x+3)(x-3)}$$

VA: $x+3=0 \rightarrow \boxed{x=-3}$
 $x-3=0 \rightarrow \boxed{x=3}$

HA: $y = \frac{x}{x^2} = \frac{1}{x} \rightarrow \boxed{y=0}$

Domain: $\{x \mid x \neq \pm 3\}$

$$5.) \quad y = \frac{x^2 + x - 12}{x^2 - 4} = \frac{(x+4)(x-3)}{(x-2)(x+2)} \quad (2)$$

$$\text{VA: } \begin{aligned} x+2=0 &\rightarrow \boxed{x=-2} \\ x-2=0 &\rightarrow \boxed{x=2} \end{aligned} \rightarrow \boxed{\text{Domain: } \{x \mid x \neq \pm 2\}}$$

$$\text{HA: } y = \frac{x^2}{x^2} = \boxed{1=y}$$

$$6.) \quad k(x) = \frac{25 - x^2}{x+5} = \frac{-(x^2 - 25)}{x+5} = \frac{-(x+5)(x-5)}{(x+5)} = -x+5$$

$$\boxed{\text{VA/HA: None}}$$

$$\boxed{\text{Domain: } \{x \mid x \neq -5\}}$$

$$7.) \quad f(x) = \frac{x^2 + x - 12}{x-4} = \frac{(x-4)(x+3)}{(x-4)} = x+3$$

$$\boxed{\text{VA/HA: None}}$$

$$\boxed{\text{Domain: } \{x \mid x \neq 4\}}$$

$$8.) \quad y = \frac{3x^4 + 4}{x(x^2 + 3)}$$

$$\text{VA: } \boxed{x=0}$$

$$x^2 + 3 = 0 \rightarrow x^2 = -3 \quad *$$

$$\text{HA: none} \rightarrow \frac{3x^4}{x^3} \rightarrow 3x$$

$$\boxed{\text{Domain: } \{x \mid x \neq 0\}} \quad \text{Divide (see next side)}$$

8 (cont)

$$\begin{array}{r}
 x^3 + 3x \overline{) 3x^4 + 4} \\
 \underline{- 3x^4 + 9x^2} \\
 -9x^2 + 4
 \end{array}$$

3

$$\therefore \frac{3x^4 + 4}{x^3 + 3x} = 3x + \frac{-9x^2 + 4}{x^3 + 3x}$$

$$\therefore \boxed{SA \rightarrow y = 3x}$$

9.) $f(x) = \frac{x+1}{x^2+1}$ ← cannot factor over \mathbb{R} .

Domain:
 \mathbb{R}

HA: $\frac{x}{x^2} = \frac{1}{x} \therefore \boxed{y=0}$

10.) $y = \frac{x^2-1}{x}$

VA: $\boxed{x=0}$

Domain: $\{x \mid x \neq 0\}$

SA/HA: $\frac{x^2}{x} = x \therefore$ no HA

$$\begin{array}{r}
 x \overline{) x^2 + 0x - 1} \\
 \underline{- x^2} \\
 -1
 \end{array}$$

So SA: $y = x$

~~0/0~~ -1